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MODELING OF THE PROCESS OF HEAT TRANSFER FROM A PLANE HEAT SOURCE OF CONSTANT STRENGTH IN THERMOPHYSICAL MEASUREMENTS

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Consideration is given to the theoretical foundations of the method of nondestructive testing of the thermophysical properties of solid materials subjected to thermal action from a plane heat source of constant strength on a bounded portion of the surface. It has been established that the evolution of the thermal process from a bounded plane heater will be analogous to the evolution of the thermal process in a plane half-space and to the processes in a spherical half-space at high values of time.

There are many methods for determination of the thermophysical properties of materials [1–3]. Investigations in this field remain topical at present, however, primarily because of the fact that the thermophysical properties of a material are found based on indirect experiments and are calculated from certain mathematical models. The accuracy and reliability of determination of the thermophysical properties largely depend on how correctly the mathematical model describes the thermal processes occurring in measurement.

In this work, consideration is given to the theoretical foundations of the method [3, 4] of nondestructive testing of the thermophysical properties of solid materials subjected to thermal action from a plane heat source of constant strength on a bounded portion of the surface.

Let us consider a model of nonstationary heat transfer from a circular plane heat source of constant strength. We obtain an expression determining the evolution of a temperature field from a bounded circular plane heater in a half-space (Fig. 1). We use the source method [5, 6]. The temperature field from an instantaneous point heat source acting in an unbounded medium will be determined by the following expression [6]:

$$T_{\text{inst}}(x, y, z, \tau) = \frac{Q}{c_\rho (2\sqrt{\pi a \tau})^3} \exp\left[-\frac{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}{4a\tau}\right].$$

In cylindrical coordinates, with allowance for the fact that $z_1 = 0$ and the body is semiinfinite, this expression is written as

$$T_{\text{inst}}(r, \varphi, z, \tau) = \frac{2Q}{c_\rho (2\sqrt{\pi a \tau})^3} \exp\left[-\frac{r^2 + r_1^2 - 2r_1 r\cos(\varphi - \varphi_1) + z^2}{4a\tau}\right]. \quad (1)$$

We can find the solution of the problem on propagation of heat from a plane heater in the form of a circle of radius R with a heat-flux density q by integration of the function (1), passing first to an elementary source:

$$dT(r, \varphi, z, \tau) = \frac{2q}{c_\rho (2\sqrt{\pi a (\tau - u)})^3} \exp\left[-\frac{r^2 + r_1^2 - 2r_1 r\cos(\varphi - \varphi_1) + z^2}{4a(\tau - u)}\right] r_1 d\varphi_1 dr_1 du\,.$$

Integrating this expression, we obtain a formula determining the law of propagation of heat from the circular heater:

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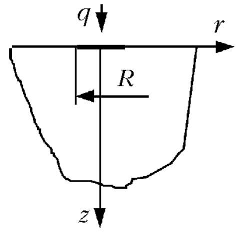


Fig. 1. Scheme for a semiinfinite body with a circular plane surface heater.

$$T(r, z, \tau) = \frac{2q}{c_\rho (2\sqrt{\pi a})^3} \int_0^\tau \int_0^R \int_0^{2\pi} \frac{\exp\left[-\dfrac{r^2 + r_1^2 - 2r_1 r \cos(\varphi - \varphi_1) + z^2}{4a(\tau - u)}\right]}{(\tau - u)^{3/2}} r_1 d\varphi_1 dr_1 du .$$

Replacing the variables and taking into account that, according to [7], we have

$$\int_0^{2\pi} \exp[x \cos(\varphi - \varphi_1)] \, d\varphi_1 = I_0(x) ,$$

we can write

$$T(r, z, \tau) = \frac{2q2\pi}{c_\rho (2\sqrt{\pi a})^3} \int_0^\tau \int_0^R \frac{\exp\left[-\dfrac{r^2 + r_1^2 + z^2}{4au}\right]}{u^{3/2}} I_0\left[\frac{r_1 r}{2au}\right] r_1 dr_1 du .$$

For the points on the z axis ($r = 0$), we have

$$T(0, z, \tau) = \frac{2q2\pi}{c_\rho (2\sqrt{\pi a})^3} \int_0^\tau \int_0^R \frac{\exp\left[-\dfrac{r_1^2 + z^2}{4au}\right]}{u^{3/2}} r_1 dr_1 du .$$

With allowance for the fact that [8]

$$\int_0^R r_1 \exp\left[-\frac{r_1^2}{4au}\right] dr_1 = 2au\left(1 - \exp\left[-\frac{R^2}{4au}\right]\right),$$

we find

$$\int_0^R r_1 \exp\left[-\frac{r_1^2}{4au}\right] dr_1 = 2au\left(1 - \exp\left[-\frac{R^2}{4au}\right]\right),$$

Let us consider the integral $\int_0^x \frac{\exp[-k^2/x]}{x^{1/2}} dx$ [8]. After a series of transformations, we obtain $2\sqrt{\pi X}$ ierfc $\left[\frac{k}{\sqrt{X}}\right]$. With allowance for this expression, we can finally write a formula determining the regularities of propagation of heat from the circular plane heater in a half-space on the z axis [5, 9] in the form

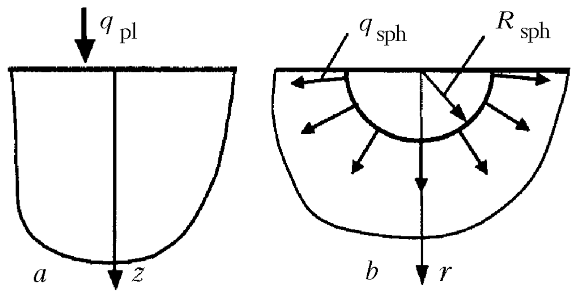


Fig. 2. Scheme of propagation of heat in plane (a) and spherical (b) half-spaces.

$$T(0, z, \tau) = \frac{2q\sqrt{a\tau}}{\lambda}\left(\mathrm{ierfc}\left[\frac{z}{2\sqrt{a\tau}}\right] - \mathrm{ierfc}\left[\frac{\sqrt{R^2+z^2}}{2\sqrt{a\tau}}\right]\right).$$

The temperature of the center of the heater ($r = 0$, $z = 0$) will be determined by the expression

$$T(0, 0, \tau) = \frac{2q\sqrt{a\tau}}{\lambda}\left(\frac{1}{\sqrt{\pi}} - \mathrm{ierfc}\left[\frac{R}{2\sqrt{a\tau}}\right]\right). \tag{2}$$

Let us consider the behavior of this function at high and low values of τ. At low τ values, we have the quantity $\mathrm{ierfc}\left[\frac{R}{2\sqrt{a\tau}}\right] << \frac{1}{\sqrt{\pi}}$, and it can be disregarded. With allowance for the fact that $\varepsilon = \lambda/\sqrt{a}$, when the values of τ are low, (2) takes the form

$$T(0, 0, \tau) \approx \frac{2q\sqrt{\tau}}{\varepsilon\sqrt{\pi}}. \tag{3}$$

For analysis of expression (2) at high τ values we represent it as

$$T(0, 0, \tau) = \frac{qR}{\lambda}\left(\frac{2q\sqrt{a\tau}}{R\sqrt{\pi}}\left(1 - \exp\left[-\frac{R}{4a\tau}\right]\right) + \mathrm{erfc}\left[\frac{R}{2\sqrt{a\tau}}\right]\right). \tag{4}$$

Then, in the domain of high τ values, we have the following dependence:

$$T(0, 0, \tau) \approx \frac{qR}{\lambda}\left(1 - \frac{R}{2\sqrt{\pi a\tau}}\right). \tag{5}$$

The average-over-the heater temperatures S at low and high values of τ [5] are determined from the dependences

$$S(\tau) \approx \frac{2q\sqrt{\tau}}{\varepsilon\sqrt{\pi}}, \tag{6}$$

$$S(\tau) \approx \frac{2qR}{\lambda}\left(\frac{4}{3\pi} - \frac{R}{4\sqrt{\pi a\tau}}\right). \tag{7}$$

Let us consider the regularities of evolution of thermal processes in plane and spherical half-spaces.

In the first case (Fig. 2a), the temperature of the bounding surface ($z = 0$) under the action of the heat source of constant strength q_{pl} will be determined by the expression [6]

$$T_{pl}(0, \tau) = \frac{2q_{pl}\sqrt{\tau}}{\varepsilon\sqrt{\pi}}. \tag{8}$$

In the second case (Fig. 2b), the temperature of the surface ($r = R_{sph}$), where the surface heat source of constant strength acts, is determined by the formula [3, 5]

$$T_{sph}(R_{sph}, \tau) = \frac{q_{sph} R_{sph}}{\lambda} \left(1 - \exp\left[\frac{a\tau}{R_{sph}^2} \right] \mathrm{erfc}\left[\sqrt{\frac{a\tau}{R_{sph}^2}} \right] \right). \tag{9}$$

At high τ values, this expression has the form

$$T_{sph}(R_{sph}, \tau) \approx \frac{q_{sph} R_{sph}}{\lambda} \left(1 - \frac{R_{sph}}{\sqrt{\pi a \tau}} \right). \tag{10}$$

Comparing expressions (3), (6), and (8) and (5), (7), and (10), we can infer that, when the values of τ are low, the evolution of the thermal process from a bounded plane heater at the initial stage will be analogous to the evolution of the thermal process in a plane half-space, whereas, at high τ values, it will be similar to the processes in a spherical half-space.

In the actual measuring experiment, the thermal process will be affected not only by the thermophysical processes of the material under study but by some other factors as well. The most important of them are heat transfer to the probe material, the heat capacity of the heater, and thermal resistances. To obtain calculated expressions in determining the thermophysical properties of solid materials with allowance for these factors we use the above analogy of thermal processes. Namely, to allow for the heat capacity of the heater and heat transfer to the probe material at the initial stage of evolution of the thermal process we will consider the problem on propagation of heat in a plane half-space, and when the values of τ are high, we will assume that the heater represents a hemisphere (or sphere).

With the aim of allowing for the heat capacity of the heater at the initial stage we solve the following problem.

Problem 1. We are given a semiinfinite body at the temperature $T(z, 0) = 0$. An infinite plane heat source of strength q and heat capacity c_h constantly acts on the bounding surface (Fig. 3a). It is necessary to find the distribution of the temperature field in this system at any instant of time. In mathematical form, this problem is written as follows:

$$\frac{\partial T(z, \tau)}{\partial \tau} = a \frac{\partial^2 T(z, \tau)}{\partial z^2}, \quad z > 0, \quad \tau > 0; \tag{11}$$

the initial condition is

$$T(z, 0) = 0, \quad z \geq 0; \tag{12}$$

the boundary conditions are

$$T(\infty, \tau) = 0, \quad \tau > 0; \tag{13}$$

$$-\lambda \frac{\partial T(0, \tau)}{\partial z} = q - c_h \frac{\partial T(0, \tau)}{\partial \tau}, \quad \tau > 0. \tag{14}$$

The solution of problem 1 for the surface layer ($z = 0$) has the form

$$T(0, \tau) = \frac{2q\sqrt{\tau}}{\varepsilon\sqrt{\pi}} - \frac{qc_h}{\varepsilon^2} + \frac{qc_h}{\varepsilon^2} \exp\left[\frac{\varepsilon^2}{c_h^2} \tau \right] \mathrm{erfc}\left[\frac{\varepsilon}{c_h} \sqrt{\tau} \right]. \tag{15}$$

In the domain of high values of $\frac{\varepsilon}{c_h}\sqrt{\tau}$, dependence (15) is transformed to the form

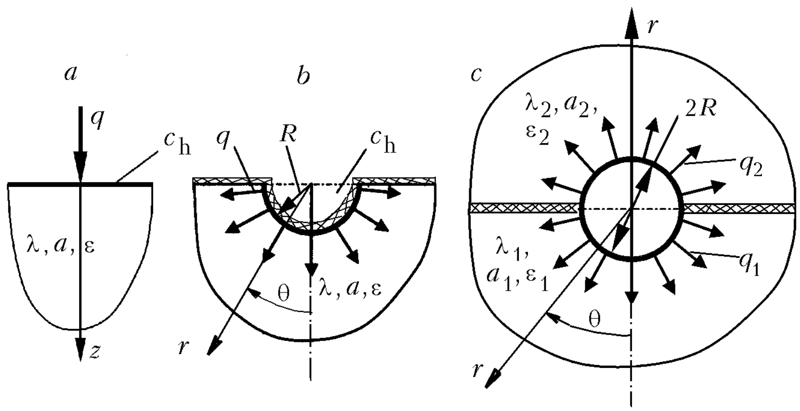


Fig. 3. Schemes to problems 1 (a), 2 (b), and 3 (c).

$$T(0, \tau) = \frac{2q\sqrt{\tau}}{\varepsilon\sqrt{\pi}} - \frac{qc_h}{\varepsilon^2}. \tag{16}$$

To allow for the heat transfer to the probe material we use the well-known formula for the distribution of heat in a system consisting of two seminfinite bodies that are in ideal thermal contact and at whose boundary a heat source of constant strength acts [6]. The temperature of the boundary of such a system will be determined by the expression

$$T(0, \tau) = \frac{2q\sqrt{\tau}}{(\varepsilon_1 + \varepsilon_2)\sqrt{\pi}}. \tag{17}$$

Thus, the heat capacity of the heater and heat transfer to the probe material at the initial stage are allowed for by Eqs. (16) and (17).

Let us consider the influence of the heat capacity of the heater on the course of a thermal process at high τ values.

Problem 2. The thermal system represents a semiinfinite body at the temperature $T(r, \theta, 0) = 0$ on whose semispherical surface (of radius R) a heat source of strength q with a heat capacity c_h acts (Fig. 3b). It is necessary to find the distribution of the temperature field in this system at any instant of time.

The temperature field in the system under study is one-dimensional and spherical; therefore, the temperature gradient in the semiinfinite body is independent of the coordinate θ.

Mathematically the problem can be written in the following form:

$$\frac{\partial T(r, \tau)}{\partial \tau} = a\left(\frac{\partial^2 T(r, \tau)}{\partial r^2} + \frac{2}{r}\frac{\partial T(r, \tau)}{\partial r}\right), \quad r > R, \quad \tau > 0; \tag{18}$$

the initial condition is

$$T(r, 0) = 0, \quad r \ge R; \tag{19}$$

the boundary conditions are

$$T(\infty, \tau) = 0, \quad \tau > 0; \tag{20}$$

$$-\lambda\frac{\partial T(R, \tau)}{\partial r} = q - c_h\frac{\partial T(R, \tau)}{\partial \tau}, \quad \tau > 0. \tag{21}$$

As a result of solution of problem 2, we write an expression for the surface temperature of a hemispherical cavity ($r = R$):

$$T(R,\tau) = \frac{q}{c_h\alpha\beta} + \frac{q(\alpha+\beta)}{c_h(\alpha\beta)^2\sqrt{\pi\tau}} + \frac{q}{c_h(\alpha-\beta)\alpha}\left(\frac{1}{\sqrt{\pi\tau}} + \alpha\exp[\alpha^2\tau]\,\mathrm{erfc}[-\alpha\sqrt{\tau}]\right) +$$

$$+\frac{q}{c_h}\left(\frac{1}{\sqrt{\pi\tau}} + \beta\exp[\beta^2\tau]\,\mathrm{erfc}[-\beta\sqrt{\tau}]\right),$$

here $\alpha\beta = \dfrac{\lambda}{Rc_h}$ and $(\alpha+\beta) = \dfrac{\varepsilon}{c_h}$.

At high values of $\alpha^2\tau$ and $\beta^2\tau$, the last expression is simplified (we use the expansion of erfc (x) in a power series at high τ values) and takes the form

$$T(R,\tau) = \frac{qR}{\lambda}\left(1 - \frac{R}{\sqrt{\pi a\tau}}\right).$$

As a result, we can infer that the heat capacity of the heater at high τ values may be disregarded.

To allow for the heat transfer to the probe material at high τ values we solve the following problem.

Problem 3. Two semiinfinite bodies at the temperature $T(r, \theta, 0) = 0$ are in contact (Fig. 3c). The contacting surfaces of the bodies are heat-insulated. A surface spherical heat source of strength $q^* = q_1 + q_2$ and radius R acts in the contact region. It is necessary to find the distribution of the temperature field in this system at any instant of time.

In mathematical form, the problem is as follows:

$$\frac{\partial T_1(r,\theta,\tau)}{\partial\tau} = a_1\left(\frac{\partial^2 T_1(r,\theta,\tau)}{\partial r^2} + \frac{2}{r}\frac{\partial T_1(r,\theta,\tau)}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T_1(r,\theta,\tau)}{\partial\theta}\right)\right),$$

$$r > R\,,\quad 0 \le \theta < \frac{\pi}{2}\,,\quad \tau > 0\,;$$

$$\frac{\partial T_2(r,\theta,\tau)}{\partial\tau} = a_2\left(\frac{\partial^2 T_2(r,\theta,\tau)}{\partial r^2} + \frac{2}{r}\frac{\partial T_2(r,\theta,\tau)}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T_2(r,\theta,\tau)}{\partial\theta}\right)\right),$$

$$r > R\,,\quad \frac{\pi}{2} < \theta \le \pi\,,\quad \tau > 0\,;$$

the initial conditions are

$$T_1(r,\theta,0)\Big|_{\substack{r\ge R\\ 0\le\theta\le\frac{\pi}{2}}} = T_2(r,\theta,0)\Big|_{\substack{r\ge R\\ \frac{\pi}{2}\le\theta\le\pi}} = 0\,;$$

the boundary conditions are

$$T_1(\infty,\theta,\tau)\Big|_{\substack{\tau>0\\ 0\le\theta<\frac{\pi}{2}}} = T_2(\infty,\theta,\tau)\Big|_{\substack{\tau>0\\ \frac{\pi}{2}<\theta\le\pi}} = 0\,,$$

$$T_1(R,\theta,\tau)\Big|_{\substack{\tau>0\\ 0\le\theta\le\frac{\pi}{2}}} = T_2(R,\theta,\tau)\Big|_{\substack{\tau>0\\ \frac{\pi}{2}\le\theta\le\pi}}, \tag{22}$$

$$\frac{\partial T_1(r,\theta,\tau)}{\partial\theta}\Bigg|_{\substack{\theta=\frac{\pi}{2}-0\\ r>R\\ \tau>0}} = \frac{\partial T_2(r,\theta,\tau)}{\partial\theta}\Bigg|_{\substack{\theta=\frac{\pi}{2}+0\\ r>R\\ \tau>0}} = \frac{\partial T_1(r,\theta,\tau)}{\partial\theta}\Bigg|_{\substack{\theta=0\\ r>R\\ \tau>0}} = \frac{\partial T_2(r,\theta,\tau)}{\partial\theta}\Bigg|_{\substack{\theta=\pi\\ r>R\\ \tau>0}} = 0\,,$$

$$-\lambda_1\frac{\partial T_1(R,\theta,\tau)}{\partial r}\Bigg|_{0\le\theta<\frac{\pi}{2}-0} = q_1\,,\quad -\lambda_2\frac{\partial T_2(R,\theta,\tau)}{\partial r}\Bigg|_{\frac{\pi}{2}+0<\theta\le\pi} = q_2\,,\quad \tau>0\,.$$

Since the temperature fields in the system under study are one-dimensional and spherical, the temperature gradient in each of the semiinfinite bodies is independent of the coordinate θ. With account for condition (22), for the relation of the specific heat fluxes $2q^* = q$ we obtain a problem equivalent to that given above:

$$\frac{\partial T_1(r,\tau)}{\partial\tau} = a_1\left(\frac{\partial^2 T_1(r,\tau)}{\partial r^2} + \frac{2}{r}\frac{\partial T_1(r,\tau)}{\partial r}\right),\quad r>R\,,\quad \tau>0\,,\quad 0\le\theta<\frac{\pi}{2}\,;$$

$$\frac{\partial T_2(r,\tau)}{\partial\tau} = a_2\left(\frac{\partial^2 T_2(r,\tau)}{\partial r^2} + \frac{2}{r}\frac{\partial T_2(r,\tau)}{\partial r}\right),\quad r>R\,,\quad \tau>0\,,\quad \frac{\pi}{2}<\theta\le\pi\,;$$

the initial conditions are

$$T_1(r,0)\Big|_{\substack{r\ge R\\ 0\le\theta\le\frac{\pi}{2}}} = 0\,,\quad T_2(r,0)\Big|_{\substack{r\ge R\\ \frac{\pi}{2}\le\theta\le\pi}} = 0\,;$$

the boundary conditions are

$$T_1(\infty,\tau)\Big|_{\substack{\tau>0\\ 0\le\theta<\frac{\pi}{2}}} = T_2(\infty,\tau)\Big|_{\substack{\tau>0\\ \frac{\pi}{2}<\theta\le\pi}} = 0\,,$$

$$T_1(R,\tau)\Big|_{\substack{\tau>0\\ 0\le\theta\le\frac{\pi}{2}}} = T_2(R,\tau)\Big|_{\substack{\tau>0\\ \frac{\pi}{2}\le\theta\le\pi}},$$

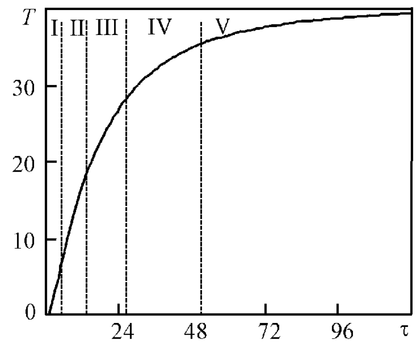
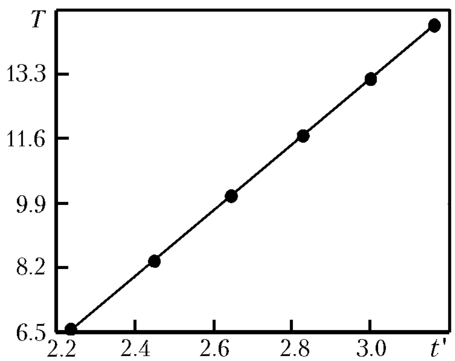


Fig. 4. Portions of the thermogram for F4K20 coke-filled polytetrafluorethylene. T, °C; τ, sec.



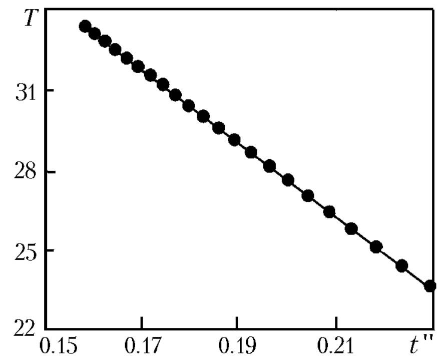


Fig. 5. Portion II of the thermogram in the coordinates $T = T(t')$. T, °C; t', sec$^{0.5}$.

Fig. 6. Portion IV of the thermogram in the coordinates $T = T(t'')$. T, °C; t'', sec$^{-0.5}$.

$$-\lambda_1 \frac{\partial T_1(R, \tau)}{\partial r}\Bigg|_{0 \le \theta < \frac{\pi}{2} - 0} - \lambda_2 \frac{\partial T_2(R, \tau)}{\partial r}\Bigg|_{\frac{\pi}{2} + 0 < \theta \le \pi} = q, \quad \tau > 0 .$$

As a result of solution of the problem we obtain

$$T(R, \tau) = \frac{qR}{\lambda_1 + \lambda_2}\left(1 - \frac{R(\varepsilon_1 + \varepsilon_2)}{\sqrt{\pi\tau}\,(\lambda_1 + \lambda_2)}\right).$$

The aforesaid allows a conclusion on the character of the thermal process occurring in the probe–heater–sample system.

Five characteristic portions can be recognized on the thermogram (Fig. 4) [3]. On portion I, the influence of the heat capacity of the heater and the "inertia" of the temperature sensor are nonlinear in character, which is difficult to allow for in determining the thermophysical properties (see the solution of problem 1 for a system consisting of two semiinfinite bodies). On portion II, the correction for the heat capacity of the heater is constant in character and the thermogram is described by an equation of the form

$$T(t') = \frac{2qt'}{(\varepsilon + \varepsilon')\sqrt{\pi}} - \frac{2qc_{\text{í}}}{(\varepsilon + \varepsilon')^2} . \tag{23}$$

As is seen from expression (23), portion II will correspond to a rectilinear portion in the coordinates $T = T(t')$ (Fig. 5). Portion III is intermediate between the second and fourth portions. On portion IV, the heat capacity ceases to affect the evolution of the thermal process. The form of the thermogram is determined only by the thermophysical properties of the material under study and by the properties of the probe. The equation describing this portion of the thermogram has the form

$$T(t'') = \frac{qR}{\lambda + \lambda'} \left(1 - \frac{R(\varepsilon + \varepsilon')}{\sqrt{\pi}(\lambda + \lambda')} t'' \right). \tag{24}$$

As is seen from expression (24), portion IV on the thermogram will correspond to a rectilinear portion in the coordinates $T = T(t'')$ (Fig. 6). On portion V, the sample ceases to be unbounded thermally, and the thermal process is considerably affected by the boundary conditions.

The performed analysis of the thermograms and the separation of five characteristic portions allow measurement of the thermophysical properties based on the analytical expressions (23) and (24) for the second and fourth portions [3, 4].

NOTATION

a, thermal-diffusivity coefficient, m^2/sec; c_h, heat capacity of the unit area of the heater, J/(K·m^2); c_ρ, heat capacity per unit volume, J/(m^3·K); $I_0[x]$, Bessel function of the second kind and zero order; q, q_{pl}, and q_{sph}, heat-flux densities, W/m^2; R, radius of the heater, m; r_1, φ_1, coordinates of the point of action of an instantaneous heat source; S, temperature average over the heater, K; T, temperature, K; $t' = \sqrt{\tau}$, $sec^{0.5}$; $t'' = 1/\sqrt{\tau}$, $sec^{-0.5}$; u, integration parameter; x, y, z, r, φ, space coordinates; τ, time, sec; ε, ε', ε_1, and ε_2, thermal activity of the material under study, correction for the thermal activity of the probe-substrate material, and thermal activities of the 1st and 2nd bodies respectively, W·$sec^{0.5}$/(m^2·K); θ, angle in the spherical coordinate system, rad; λ, λ', λ_1, and λ_2, thermal-conductivity coefficient of the material under study, correction for the thermal conductivity of the probe-substrate material, and thermal-conductivity coefficients of the 1st and 2nd bodies respectively, W/(m·K); τ, time, sec. Subscripts: 1, material under study, the 1st body; 2, probe-substrate material, the 2nd body; h, heater; pl, plane; sph, spherical; inst, instantaneous.

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